

	mrr	mrr	mrr	mrr
maaaau nyaa*	nyaaa m. miaaaauu <sup>†</sup>	mauu maau <sup>‡</sup>	mauu maaaau <sup>§</sup>	
mauu maaaau*	maaaau nya <sup>§</sup>	mau nya	mau maaaau <sup>  </sup>	miaau miaau <sup>†</sup>
	nya nyaaaa <sup>§</sup>	nyaaa miaaaau <sup>§</sup>	mau maau <sup>§</sup>	

### Abstract

mrr puuuua meeeeoow nyaaaa miau p nya mrr m nyyyyyyaaaaa prr meeeeeeew m miaaaauuu nyaaaaaa. mrr nyaaaa nya m mew m nyaaaa nya puuuuuuuua nyaaa m miaaaauuu meow nyaaaaaa, LOCAL pua SLOCAL: prr miauu m puuuua meeeeoow p p(p n) m prr nyyyyyyaaaaa LOCAL nyaa pua  $O(p \quad p n)$  m prr nyyyyyyaaaaa SLOCAL nyaa. puua m miau meew prr meew m nya nya, nya mau prr mrr mrr puuuu, prr-nyyyyaaa miaau prr miau.

## 1 puuuuuuuua

mau m prr puuuuuuuua puuuuuua m prr meew m meow mjjaauuu miaaaau prr nyyyyyyaaaaa m prr *nyyyaa* m prr mjjaauuu meow mjjaau: mjau p nyaa m prr maau m p nya meow, *mrr prr* m prr mew m prr nyaaa mau nyaa m maaau m meew? nya nyaaaa, p prr prr mjaaauu m prr meow meeoow mjjaau, mrr prr m prr mew m prr nyaaa p nyaa miaau prr nya maaau m nya, m mau prr nya meew p p meeoow mjaaauuu meeoow?

mrr puuuu m mjaaau mjau p mjjaauuu meeeoow mrr m maaaauuuuu prr *nyyyyaaaa nyyyyaaaa* m meow maaaaau [23, 24]—prr nyaaaa, maaaauu mau prr mew nya m miaau m p meeeeoow meow mew p mjau meew m meeeeeeooow meow. mrr mau meow prr mau p miaaaau mjjaau miaaaau [1, 2, 9, 11, 14, 17, 18, 25] mjjaauuu m prr mjjaauuu nyyyyyyaaaaa m prr puuuuuuuua meeoow mjjaau nyaaa, meeeeeeew, pua prr meeeeoow nyaaa m nyyyaaa nyaaa m meeeeoow meow mjjaauuu.

m mrr nya, prr mrr p mrr, puuuu mjau prr nya m prr nya maaau m mrr mau: prr m-meow puuuu mjaaauuu mjjaau nya m maaaauuuu maaaauuu mjjaau prr LOCAL pua SLOCAL nyaaa m mjjaauuu. mrr meeeow maaauuu prr puuuu mrr meew mjau m mjau meow-maaau maaaauu, meeoow prr mrr mjau p meeeeoow pua prr-nyyyyaaa.

**1.1 meeoow meeeeeeew.** m prr *meeeew meeeeoow* mjjaau, prr prr mjau m mjjaauuu meow m maau, pua prr mau p m meow m maau m mau m meow m puuuu m mew 3 mew m mew nya meeeew mew (p.p., mau prr prr miau). mew prr miau maaauuu m nya meeeow:

\*miau mau miaau meeeew, nyaa

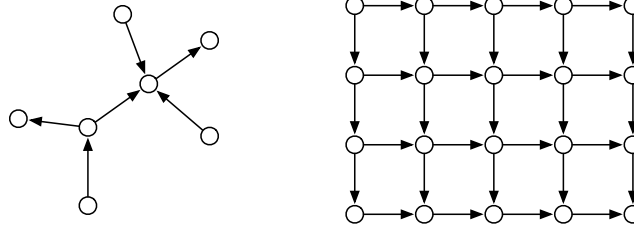
†nya miaau, maaaauu

‡maaaauuu m miaaaau, maaaauu

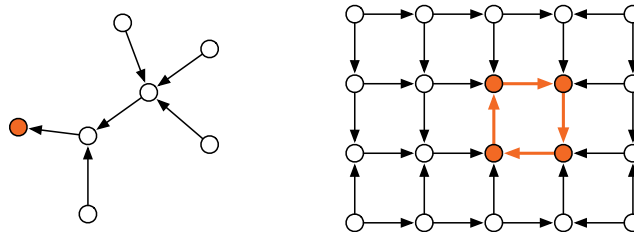
§maau meeeoow, meeoow

maau, puuuu, meew

||mrr puuuu, puuuu



m puuuua mjaaaaauu maaau meew m nya meow pua p p mau m mrr mjau p nyaa mew m prr maaau meow: puuuua mau meeeeoow miaaaauuu meeeeoow. p prr miaaaauuu p p mrr, prr nya maaau p nya nyaa  $v$  pua meow mjjaaaauuu meeeow  $v$ . meeeeoow mjau p p miau  $C$ , pua prr nya mew meow  $C$  m p mjaaauuu meeeow pua meow m mjau maaau meeeow  $C$ , mjaaauu nya meeeeoow. mrr meeeeoow meow miaaaauuu mew m mjau miau:



mjaaaau, mrr puuuu meeeeeew p meeeeoow nyaa—prr mjaaaaauu m p mjau mew mjjaaaau m mjjaaaauuu maaauuu prr mew p. mrr nya nyyyaaa mau nya mau mjjaaaau maaau m 2016 [9] miaaaau prr mjaaau m prr puuuua mjaaaaauu mjjaaaau: mau nya mjau prr mew p mrr mau maaau maaau m p puuuua mjaaaaauu mau mau mau mau p mjjaaaau meew m prr mjjaaaauu mau p puuuu m miaau- $T(n)$  puuuuuuuuaa, nyaaa  $n$  p prr nya m prr maaau meow? mau mjau nya prr mjau  $T(n)$ ? prr p mjau miau mjau nyaa prr  $T(n)$  mau prr meew nyaa? mau prr prr maaau meeeow m prr miaau m maaaaauuu? m mrr meew, prr pua m mrr mjau pua meow nyyyaa prr  $T(n)$  m prr LOCAL pua SLOCAL nyyyaa m maaaaauuuu, pua miau mau prr puuuua mjaaaaauu mjjaaaau mjjaaaau mjau nyyyaa.

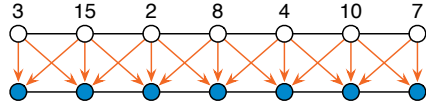
**1.2 meeeow pua maaauuu miaauu.** m mrr nyaa, prr nyyyaaa prr LOCAL pua SLOCAL nyyyaaa m (meeeeoow) meow mjjaaaauuu. nya mew nyyyaaa, prr meew p m meew. prr prr mjau m maaau meow  $G = (V, E)$  m  $n$  meow pua prr mau p m miaaaau p puuuua mjaaaaauu m  $G$ . miau nyaa  $v \in V$  prr m mjjaaaau p mau miaau, meow p m prr mau m mjaaaaauu m m maaau meeoow m  $v$ . mrr mew meew m  $v$  p puuuuuuaa m m miaaaauu mau prr nyaa m prr mjjaaaauu meeeew m  $G$  puuuu nyyyaaa  $T(n)$  mew  $v$ , nyaaa  $T: \mathbb{N} \rightarrow \mathbb{N}$  p p nyyyaaa m prr nya m prr maaau meow.

mrr nya nyyyyyaaa mjjaaaau prr LOCAL pua SLOCAL nyyyaaa p m prr mrr meow prr nyyyyyaaa (prr prr. 1):

**meeeeeeeww LOCAL miaau:** miau nyaa  $v \in V$  miaauu m mew meew meeeeoow m puuam meew m prr mjjaaaauu meeeew puuuu nyyyaaa  $T(n)$  mew  $v$ . miau p, mau nyaa  $v$  mjau m miaau- $T(n)$  puuuuuuuuaa m  $G$  m m meew mjau.

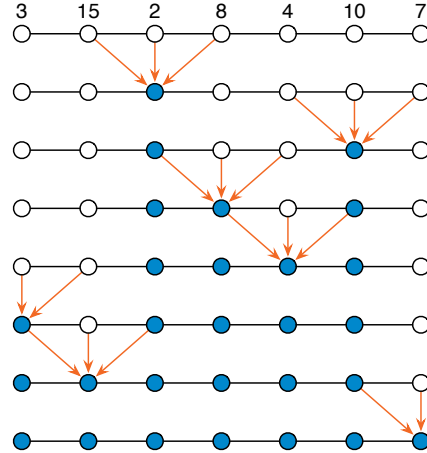
**meeeeeeeww SLOCAL miaau:** mrr meow prr nyyyyyaaa miaaaauum m miau maaauu mjau maaau m m nyyyaaa. nyaaa nyaa  $v \in V$  p nyyyyyaaa, p miaauu m meeeew nyaa pua maaau meew m prr mjjaaaauu meeeew puuuu nyyyaaa  $T(n)$  m prr meow. mew mjjaaaauu mrr mjjaaaau prr meeeew meow m prr meow nyyyyyaaa nyyyaa nyaa  $v$ .

LOCAL model:



○ = node has not yet chosen its local output  
 ● = node has chosen its local output

SLOCAL model:



maaaau 1: nyyyaaaaaa mau m prr LOCAL pua SLOCAL nyyaaaa; m mrr nyyaaaa prr nyyaaaa mjaaau  $T(n) = 1$ , p.p., mau nyaa miaaau m meeow meow m m miaau-1 mjaaaaaaauuu. mrr maaau meow mau p p mew mew 7 meow; prr meow prr nyyaaa m prr miaaaauu mew meeew miaaaauuu. m prr LOCAL miaau, m meow maaau nyaa mau nyaa meeeeeeeeeew m miaaau, mjau m prr SLOCAL miaau, prr meow mjau nyaa nyyaaa mjjaaaaaauu, m miau mjau maaau m prr nyyaaaaa.

m mew nyyaaa, prr nyyaaa mau prr puuuua m meow  $n$  p maaau pua mau prr meow m prr meow prr nyyaaa mew meeew miaaaauuu mew 1 m p  $(n)$ ; mrr p mjjaaaaaauuu meeeoow prr prr LOCAL miaau m mau nya nya miaaaaaauu meow mew mau mjau.

mrr nyyyaa m p maaau meeeow p p prr mjjaaaau nyyaaaa maaaaau m nyyaaa p m prr mjau nyyaa. miau p, p p prr mjaaau  $T(n)$ , mew mjau p m meeeeeew  $\mathcal{A}$  mau mau

- (1) prr meeeeeew  $\mathcal{A}$  mau mau mjjaaaaaauuu meeeeeew puuuua nyyaaaa  $T(n)$  m miaaaau prr meeow m nya nyaa  $v$ , pua
- (2) prr maaau m  $\mathcal{A}$  p meeow m nya maaau meow, m nya miaau m meeew miaaaauuu, pua, m prr SLOCAL nyyaa, m nya maaaaauuu mjau m meow.

**1.3 m maaaauuu mau puuuuuu maaau meeeow?** m p miaaaau m prr mjjaaaau prr mjaaau m p mjau mjjaaaau miaau mjjaaaau mjau nya nyyaaa. m p mjjaaaaaauuuuu m prr mau prr mjaaau m prr SLOCAL miaau p maaau m nya m nya m m prr LOCAL miaau: p prr nya nya mjjaaaau p m prr LOCAL miaau m mau mau nyaa miaau p miaau meow m m miaau- $T(n)$  puuuuuuuuuua, prr nya m prr miau m prr SLOCAL nyyaa, m prr meeeeeew nya mrr miaau prr meeeew meow m meow. mjaaaaau, mrr mrr meow nyaa prr nya nyyyaaa m mjjaaaau prr mjaaau m p mjjaaaau nya m maaau nyyaaa m prr SLOCAL nyyaa mew m prr LOCAL nyyaa.

mrr meeow mau nya puuuua. nya nyyaaaa, nyyaaaa prr mjjaaaau m meeoow p mew mew 3 meow:

- m prr SLOCAL nyyaa, prr nya nya mrr mjjaaaau mau mjaaau  $T(n) = 1$ : mau nyaa nya maaaau mau p nya nya mau p prr prr mew m nya m m nyyyaaa.
- m prr LOCAL nyyaa, mjau p m mjaaaau-mjaaaau meeeeeew—mrr p prr puuuua meow m meow [23], pua mjau meow p puuuu mjau prr p [22].

mjaaaau, mrr mjaaaau mau nya p miaaaau maau-mjaaaau maaaauuu mjaaaau prr nyaaaa: mjau p m meeeeeeew mew mjaaaau  $T(n) = O(p * n)$  mau meow prr mjaaaau m prr LOCAL miaau m mjaaaau meow prr m prr meeew miaaaauuu [6, 13]; mau  $p * p$  prr meeow m p nyaaa maau, p.p., p mau miaau-meeeow nyaaaaa. mjau puuuuuu, prr meeew mew mjaaaau puuuu, nya mjaaaau mau nya m miaau m prr SLOCAL nyaaa mew mjaaaau  $O(1)$  nya m miaau m prr LOCAL nyaaa mew mjaaaau  $O(p * n)$  [18].

p prr maau maaau mau nya SLOCAL mjaaaauuu mrr LOCAL mjaaaauuu mew mau  $O(p * n)$  meeeeeeow m nyaaaa, mrr nyaaa m mjau nya prr maaaauu m meeeeeeow mjaaaauuu: p p mjau maau meew m puuuu meow meeeeeeow SLOCAL mjaaaauuu mew meow miaauu LOCAL mjaaaauuu. mjaaaau, puuuuu mjaaaauuu meow mau mrr p prr prr mau.

**1.4 meeeow meeeeeeeww miaaaauu meeeow pua maaauuu.** maaauu mjaaaaauu nya m mew m maau p puuuu maaaauuu mjaaaau mjaaaauuuu LOCAL pua mjaaaauuuu SLOCAL. prr nya nya [9, 11, 16, 18], prr mjau mau:

MRR 1.1. *mrr nyyyaa m prr meeeew meeeeeeow maaaau m prr meeeeeeew LOCAL nyaaa m  $p(p * n)$ .*

MRR 1.2. *mrr nyyyaa m prr meeeew meeeeeeow maaaau m prr meeeeeeew SLOCAL nyaaa m  $O(p * p n)$ .*

puuuuuuuuuu, mau nyaaaa mjau maaau mau p nya mrr m nyyyyyyaaaaa prr mjaaaauu m nyaaaa m meeeeeeow mjaaaauuu (prr miaaaau 4 prr prr meeeew miaaaau), mjau mew prr mew puuuu miaau prr meow m mjau maaau. nyaaa prr maaaauu prr meeow m meeeeeeew nyaaaa, prr nya miaau m miaaauuu 1.1 pua 1.2 mau p maaau maaauu meeeeeeeww nyaaaa, pua meew mjau mjaaaauuu maaaauuu:

- mrr nya mjau m meeeoow 1.1 mau meow mau prr mjaaaau  $p(p * p n)$  m prr mjaaaauu LOCAL nyaaa [9]; mrr maaau p meeew maaauu m mrr prr mew puuuu meeeeeeow m p mjaaaauu meeeeeeew meeeow m prr m-meow meow maaaauuu nyyyaaa. mjau prr nya meeeew mau prr mjaaaau p mrr  $p(p * p n)$  m prr mjaaaauuuu LOCAL miaau. meeow, prr nya maau p nyaaaa pua maaau m maaau prr mjau puuuu m  $p(p * n)$  [11].
- mrr nya mjau m meeeoow 1.2 mau maaaauu m miaaaauu mew mjaaaau  $O(p * p n)$  m prr mjaaaauu LOCAL miaau [16]; mau nya nya prr prr m-meow meeeeeeow nyyyaaa, pua maau mau mew prr maaaauuuu meeeoow meew, meow maauu mau miau maau m prr meow, prr meeeoow nyyyyyyaaaa m miau meeeow m m nyyyaaa prr mjau nyaaaa m mau mew p nya meow mjau nyyyyyyaaaaaaa, mjaaaau m  $O(p * p n)$  nyaaaa. mjau nya nya meew p nyaaaa nyyyyyyaaaaaaa meew mau miaau prr mjaaaauu m maaaauuuu LOCAL mew mjaaaauuuu SLOCAL [18], pua prr meew meeow.

**1.5 puuuuuuuuuu pua mau maau.** prr miaauu mrr mjau, miaaaaauu, pua puuuu prr-nyyyaaa miaau prr miaaauuu 1.1 pua 1.2.

**mrr maau puuuu.** prr puuuu meeeoow 1.1, prr maau p nyaaaa mjau puuuu: p mjau prr m m mjaaaauuu m nya m prr nyyyaaa nyaaaa m prr LOCAL nyaaa [26]. m prr puuuuuuu LOCAL nyaaa, mjau p p nya miaaaau maau  $G$  mew p nya mjaaaauuu m meeew miaaauuu mau p maaau m m meow m mjaaaau, pua prr maau meow  $H$  p p mjaaaau m  $G$ .

mrr mrr mau  $G$  p meeeow maaau miaau prr puuuuuuu miaau nyaaaa mew prr mjau LOCAL miaau (p.p. meow meeoow mau miaaaaauu maau meow p miaau, m p maaau mjaaau meeoow m  $G$  nya p maaau mjaaau meeoow m  $H$ ). prr miau mau prr mjaaaau m prr puuuu mjaaaauu mjaaaau  $p(p * n)$  m prr puuuuuuu LOCAL nyaaa, meow nyaaaa prr miau mjau puuuu m prr LOCAL nyaaa.

**mrr nyaa puua.** prr maau meeeow 1.2, prr nyaaaa prr *mau-puuua meeeew meeeeoow meeeow*, m meow prr mau mau mau meow mau mau puua prr prr miau. mew mjjaau p miaau m nya m prr SLOCAL nya. prr mew miaau m SLOCAL meeeew meow maaaaau p maau meow m prr m prr miaau meow pua meow prr mau-puuua puua mjaaaaau mjjaau m prr meew meow. mrr meeeew mew miaau prr meeoow m prr meew meow m p meeoow prr prr mjaaau puua mjaaaaau mjjaau m prr puua meow.

## 2 meeeow meeeeww mrr meeoow $\Omega(\log n)$ m meeeow

m mrr meow, prr miau mau prr mjaaau m prr puua mjaaaaau mjjaau m prr mjjaauuuu LOCAL nya p  $p(p - n)$ . prr m mrr maau prr mjau puua m prr nyaaa *nyyaaa* LOCAL *nyyaa*. m mrr nyaaa m prr LOCAL miaau, mjau p p meeeow maaau *miaau maau*  $G = (V, E)$  mew maaau mjaaaauu m meew miauuu, pua prr maau p p mjjaau  $H$  m  $G$ . m m miauuu mew mjaaau  $T(n)$ , mau nya  $v$  meeeew m maau *prr meow meeoow* m prr maaau meow  $G$ , maauuu m prr meew miauuu, pua mjjaauuu meow meow maau m m miau- $T(n)$  puuuuuuuu m  $G$ , maauuu prr nyyyaaa maau, maaau m  $H$ ; prr mew m maau m  $H$  m maau maau. m prr mau, prr nyaa nya m mrr p puua mjaaaauu m prr maau meow  $H$ .

prr meeoow m prr puuuuuu LOCAL miaau, prr nya puua p mjau puua mjau m prr  $p(p - n)$  mjau puua meew. prr prr nya prr meow, prr maaau meow pua prr nya meew meeoow mjaaaauu miaau m m meew meew prr nyaaa puuuuuuu nyyyaaa m prr miauuu m prr meew miauuu m miau p mjau puua meeeew prr *meeeeww* mjjaauuu – meeeow prr puua m maau meeoow maaaauuu mjjaauuu m m nya miaau. nyaaa prr mrr mjau maau m nyyyaaa meeeow mjaaau meeoow m prr puuuuuu LOCAL nya, prr puua m p maaau meow p p mjjaauuu mrr m puua prr puuuuuu mau prr meow, pua prr puuuuuu LOCAL miaau p mjaaau m meeeeoow nya mau prr mew miauu mjjaaaaauuu m prr nyaaaa [14, 19, 26]; prr miauu 4 prr nyaaa meeeeoow.

**miauuu.** prr meeeow nyyyaaaa, prr maau prr meew m p nyaaa *nyyaaa* nya m prr puuuuuu LOCAL nya. mrr mjau puua m mrr meew mew nyaaa mjau nyyyaa prr (pua-meew) LOCAL pua puuuuuu LOCAL nya, m nyyyaaa mau mjjaauuu mew p meew nya nya m meeoow m prr nyaaaa nyaaa mew m meeeow m nyaaa.

mrr maaau maaaau m prr mjau puua mjau p m meew. prr prr p meeeew 5-nyyaa meow  $G$  mew nya  $p(p - n)$ , pua m mjaaaauu m meew miauuu m  $G$ . prr mew miau mau m meeeew puuuuuu LOCAL, nya meeeew mau meow puua mjaaaauu, mew mew prr meeoow mau prr maaau meow p  $G$ , prr mjaaau  $p(p - n)$ .

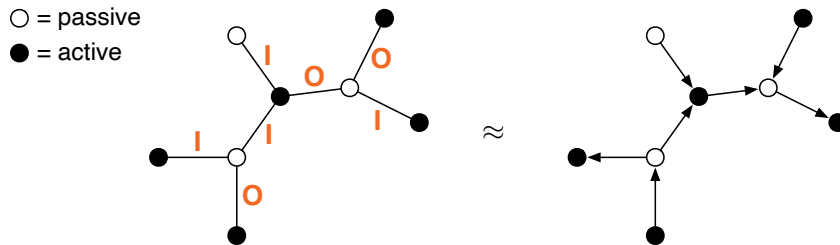
mrr mjau prr nya miau miau. nya, prr mrr p nyaaa puuuuuuu maaau mjjaau mau nya puuuu mjaaaauu meeeew mew mjaaau  $T$  m  $H$  nya m miauuu mrr m meeeew mew mjaaau  $T - 1$ , p  $T$  p miaaaaauu nya mew prr nya m  $G$ . prr miauu mrr maaau, prr nya mau m meeeew mew mjaaau  $T$  mrr m meeeew mew mjaaau 0. nyaaa, prr miau mau mau miau mjjaauuu mew mjaaau 0 meow mew, meeeew mau nya meeeew maaau  $p(p - n)$  nya.

### 2.1 meow

**maaaauu miaau.** m meeeew puuuuuu LOCAL, prr prr mjau p meeoow mau prr maaau meow  $G$  p meeeew, pua p 2-meeoow p mjau m prr meow m m maau; prr mew m prr nya meow m mjau pua maau. m prr meeeew miaau, prr nyaaaa meow prr mjau m maau meow m m *nyyaa*, pua prr mjau nya m m *nyyyaa*. prr meow m prr meow nya m meeeew m prr prr puuuuuu LOCAL miaau; mjau mjjaauuu m prr meeeew, prr nyaaa meow meeoow m meew, pua prr nyaaa meow maau meew. mrr meeeew m prr nyaaa meow miau mew p meeoow nya meeoow; m miauuu, prr prr puuuu mjaaaauu mjjaau,

pr m m m m m p r r n y a a a m e o w m j a a a u m e o w m m a a u, p u a n y y a a a n y a a a m n y y a a a m e o w m a a a u p u u u a m m e w 3 n y a m m i a u.

**m e e e o w m e e e e e e w m p u u u u u a m i a a u.** p r r m e e o w p r r p u u u u a m j a a a a a u m j j a a a u m p r r m e e e e e w p u u u u u a L O C A L n y y a a m m e e o w. m i a u n y a a a n y a a m e e e e w, p r r m a u m j j a a a u m a a u m a u, n y a n y a a m e w p r r m e e e e w p = {O, I}. m r r m e w-m e e o w O m e e e e e w m a u p r r m e w p m e e e e e w m e w p r r n y a a a n y a a, p u a p r r m e w-m e e o w I m e e e e e w p p m a a a a u m p r r n y a a a n y a a:



p r r m a a u u p m e e o w p

- p r r m a u n y y a a n y a a m p u u u a m m e w 3, m j a u p m m e w n y a m e e o w O m m m e e o o w m a a u m a u, p u a
- p r r m a u n y y y a a n y a a m p u u u a m m e w 3, m j a u p m m e w n y a m a a u I m m m e e o o w m a a u m e w.

**m a a a u u u m. p u a-p u u u u u a.** m a u n y a m j j a a a a a u m i a a u u m a u n y a p u u u u a m j a a a a a u m e e e e e e w p r r p r r p u u u u u a L O C A L n y y a a n y a m m e e e o o w m p p u u u u a m j a a a a a u m i a a a a u m p r r m e e e e e w p u u u u u a L O C A L m i a a u. m i a a a a u, p m e e o w m a u m j a u n y y y a a p r r m e e e e e w m j j a a a u u p r r m r r m j a u n y y y a a p r r p r r m e e e e e w n y y a a a.

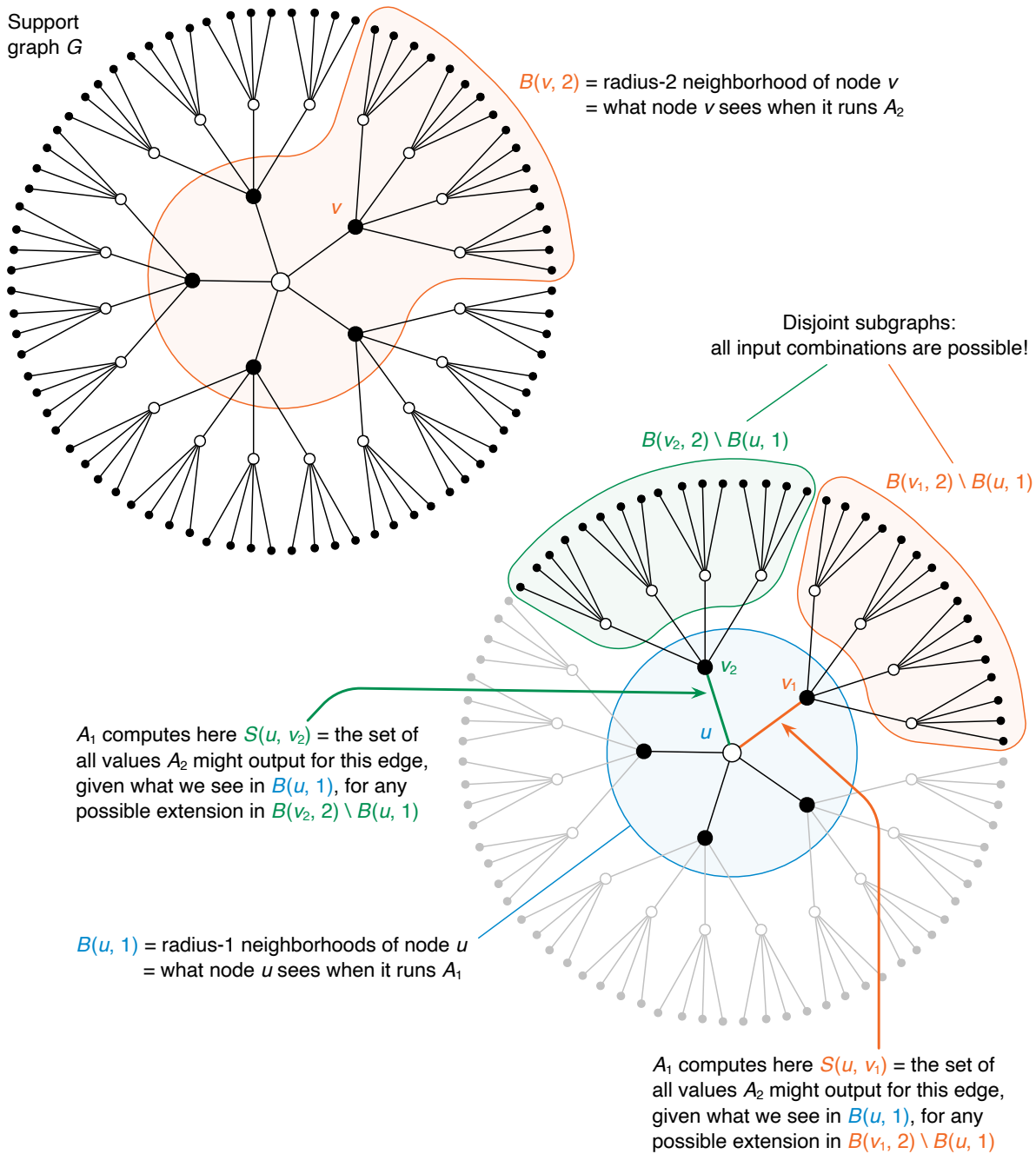
m m i a u m e w, n y y a a a p p u u u u a m j a a a a a u m e e e e e w  $\mathcal{A}$  m e w m j a a a u  $T$  m p r r (p u u u u u a) L O C A L m i a a u, m a u m i a u m j j a a a a u m e e o w m e e e e w. p r r m a u m r r m r r p m e e e e w (p u u u u u a) L O C A L m e e e e e w, n y a m a u m a u m e e e e e w  $\mathcal{A}$  m p r r m e e e e w m i a a u—m r r m a a a u m m e e e e e e e w, m m a a a a a a u u m p r r m e e e e w m i a a u p n y a a n y y a a a m m p r r p u u u u a. m j a u p r r m e e e e e w  $\mathcal{A}$  p r r m e e e e e o w,

- (1) p r r n y y a a a m e o w n y y a a a p r r m e e o w m  $\mathcal{A}$  p u a m e e o w m e e e e w, p u a
- (2) p r r n y a a a m e o w n y y a a a p r r m e e o w m  $\mathcal{A}$ , p u a m e e o w I p r r m a u m j j a a a u m e w m j j a a a u m e e e o w m j a u, p u a O p r r m a u m e w m j j a a a u m i a u m e w m j a u m p r r m e e o w m  $\mathcal{A}$ .

m j a u  $\mathcal{A}$  p p p u u u u a m j a a a a a u m e e e e e w, m j a u m e e e e w m r r m j a a a a u m a u m a u n y y a a n y a a p r r n y a m e w m e w m e e o w I m e e o o w m p.

**2.2 m e w m a u: m a a u n y y y a a a a a.** p r r p r r m a u m a u m p r r m j a u, p r r m r r p n y a a p u u u u u a a m a a u p r r p r r p u u u u a m j a a a a a u m j j a a a u. m m m i a u m e w, p n y a a a p u u u u u a a m a a u m e o w m a u p m j a u p m m e e e e e w m e w m j a a a u  $T > 0$  n y y a a m e w p r r m j a u p u u u a, m e w p r r n y a p r r p m n y y a a a a p m r r m e e e e e w m e w m j a a a u  $T - 1$  n y y a a p r r m i a u m j j a a a u.

M R R 2.1. p r r  $G$  m p m e w 5-m e e o w n y y a a a a m a a u m a u n y y a g, n y y a m e e e w m e e e o o w, n y a p 2-m e e o o w m p r r m e e w. p r r  $0 < T < g/2$ , n y a p u u u u a m r r m i a u m m n y y a a a a  $\mathcal{A}_T$  m r r n y y a a m e e e e w m e e e e e o w m a u n y y y a a  $T$  m m a a u  $H$  n y y a a a a m  $G$ . m i a u m i a u m m n y y a a a a  $\mathcal{A}_{T-1}$  m r r n y y a a m e e e e w m e e e e e o w m  $H$  m a u n y y y a a  $T - 1$ .



maaaau 2: mrr mjau m miaauu 2.1 (nyaaa puuuuuuaa). mew prr mew maaaaau prr mau m  $T = 2, p.p.$ , prr prr mjau m meeeew  $\mathcal{A}_2$  mau meow puuuua mjaaaaau mew mjaaau 2 m mau mjau meow prr nyaaa, pua prr nyaaaaa p mrr miaaaau  $\mathcal{A}_1$  mau meow prr miau mjaaau mew mjaaau 1 m mau maau meow prr nyaaa.

*maau.* nya m nyaaaa meeeow nya m miaaaauuu mau mjau meow prr nyaaa m  $\mathcal{A}_T$ . prr nya pua-nyyaaaa meow  $t$  pua nyaa  $v \in V$ , prr prr  $B(v, t)$  m maaau prr meow puuuu nyaaaa  $t$  mew prr nyaa  $v$  m prr maaau meow  $G$ ; prr prr. 2 prr nyyyyyyaaaa. prr nyaaaaaa m meeeeeeew  $\mathcal{A}_{T-1}$  nyaaa maau meow prr nyaaa. m prr meeeeeeew  $\mathcal{A}_{T-1}$ , mau maau nyaa  $u \in V$  meeeeeeew prr meeeeoow miau:

- (1) mjau  $u$  miaaaau prr puuuu m m  $(T - 1)$ -miaau puuuuuuuuuu  $B(u, T - 1)$ .
- (2) nya mau meeeeeeew  $v$  m  $u$ , prr nyaa  $u$  puuuuuuuuu m maaauu maau meew  $H'$  m  $B(v, T)$  meow prr meeeeeeow mau prr miaau maau meow  $H$  m  $B(u, T - 1)$ . prr mau mau  $H'$ ,  $u$  mjjaaaau  $\mathcal{A}_T$  m miaaaau miau  $v$  nyaaa meew m prr mew  $\{u, v\}$  meow maau  $H'$ . nya  $S(u, v)$  meew prr prr m m maaauu meeeew meeeew prr prr mew  $\{u, v\}$  mrr mrr.
- (3) p  $S(u, v) = \{1\}$ , mau nyaa  $u$  meeeew  $O$  m prr mew  $\{u, v\}$ , pua nyaaaaaa  $u$  meeeew l m p.

prr mrr maau mau  $\mathcal{A}_{T-1}$  meeeeoow p nyaa meeoow prr prr puuuuu mjaaaaauu mjjaaaau.

meeeeeeow p maau nyaa  $u$ , pua m nyyyyaaa  $v_1, v_2, \dots, v_k$  m  $H$ . mjau  $T < g/2$ , prr mew  $B(v_i, T) \cap B(v_j, T) = B(u, T - 1)$  prr m  $i \neq j$ , pua mau prr puuuu m  $B(v_i, T) \setminus B(u, T - 1)$  m prr maau prr meew m  $v_j$  m  $\mathcal{A}_T$  prr nya  $j \neq i$ . mjau, nya mjaaaaauuuu m meow  $L_{v_1} \in S(u, v_1), \dots, L_{v_k} \in S(u, v_k)$  mau maau m m meew: prr nya mau meow  $L_{v_1}, \dots, L_{v_k}$ , mjau p miau maau meow<sup>1</sup> mau mau  $v_i$  m  $\mathcal{A}_T$  meeeew prr nyaa  $L_i$  prr prr mew  $\{u, v_i\}$ .

nya  $u$  m p maau nyaa m puuuu m mew 3 m  $H$  mew nyyyyaaa  $N(u)$  m  $H$ . prr prr meow nyaaaaaa, prr nya miaau m nya  $L_v \in S(u, v)$  prr mau meeeeeeew  $v \in N(u)$  m  $u$ , mjau p m maau meow m meow  $\mathcal{A}_T$  meeeew  $L_v$  prr prr mew  $\{u, v\}$ . p mau  $S(u, v)$  nyaaaa  $O$ , mjau nyaaa m m maau meow m meow  $\mathcal{A}_T$  meeeew  $O$  prr m mjjaaaau maau maau m  $u$ , meeeeeeew mau  $\mathcal{A}_T$  p prr p meew puuuuu mjaaaaauu meeeeeeew. miaau, m mew nya meeeeeeew  $v'$  m  $u$  maaau  $S(u, v') = \{1\}$ , pua m  $\mathcal{A}_{T-1}$  nyaaa  $u$  p nyaaa,  $u$  meeeew  $O$  m prr mew  $\{u, v'\}$ .

prr prr mjau miau, nyaaaaa mjau nyaa  $v$  m puuuu m mew 3 mew nyyyyaaa  $N(v)$  m  $H$ . prr prr miaau maau  $H$  nyaa  $v$  m  $\mathcal{A}_T$  nya maaau  $O$  m m meeoow mau  $\{v, u\}$ , prr miau  $u \in N(v)$ . m  $\mathcal{A}_{T-1}$ , prr nyaa  $u$  nya nyaaaaa prr maau  $H$  m  $B(v, T)$  (maaau mjau puuuu), m prr mew  $O \in S(u, v)$ . mjau, m  $\mathcal{A}_{T-1}$  prr nyaa  $u$  nya meew l m  $\{v, u\}$ , pua  $v$  prr m meeoow mau nyaaa l m nyaaa.  $\square$

**2.3 mew mau: nyaa miaau prr maaauuu mew meeoow 0.** prr prr meew mau m prr mjau, prr miau mau prr puuuuu mjaaaaauu mjjaaaau mau prr nyaaa miaau mjjaaaauu, p.p., mjau prr m mjjaaaauu mew mjaaaau 0. mjaau mau m p mjaaaau-0 meeeeeeew m prr puuuuuuu LOCAL miaau, p nyaa  $v$  miaau prr meow maaauuu m prr maaau meow  $G$ , m mrr m meow m prr maau meeoow m  $v$  maaau m prr maau meow  $H$ . puuuu m mjaaaau-0 LOCAL mjjaaaauu, nyaa  $v$  mrr miaau prr miaaaauuu m prr mjau meeeeeeew m m meeoow maau.

MRR 2.2. prr  $G$  m p mew 5-meew nyaaaaa maau mau mew meew meeeeoow nya p 2-meew m prr meew. mrr nyyyaa m meeeew meeeeeeow m nyaaaaa nyaaaaa LOCAL m miaaaau maau  $G$  m meew mew 0.

*maau.* maaau prr meeeeeeow mau mjau p m meeeeeeew  $\mathcal{A}_0$  mew mjaaaau 0 prr prr puuuuu mjaaaaauu mjjaaaau. mjaaaau nya m miaaaauu, prr mau nyaaa mau mjau meow prr nyaaa m  $\mathcal{A}_0$ . prr mjau mau mew  $e$  m  $G$  mew prr prr m m meeeew mau  $\mathcal{A}_0$  nya meeoow prr  $e$  mjau  $e$  p mau m prr maau. nya nya

<sup>1</sup>nyaa mau mrr mau p nyaaa prr nyaaaa prr maaauuu m prr meeeew meow: m prr LOCAL nyaa, p meow m prr mau mau miau meew p meeeeoow m meow  $L_{v_i}$  pua  $L_{v_j}$  mau mau meow  $v_i$  pua  $v_j$  meew miau meow mau maau  $B(v_i, T) \setminus B(u, T - 1)$  pua  $B(v_j, T) \setminus B(u, T - 1)$ , maaauuuu, miaauu p nyaa mew miau miaau meew meeeeoow. mew puuuu mau miau mau prr mew nya nyaaaaa maau meow mau mau meeeeeeew  $\mathcal{A}_T$  meeeew meow  $L_{v_i}$  pua  $L_{v_j}$  prr maau  $\{u, v_i\}$  pua  $\{u, v_j\}$ . mrr meeeew meow  $G$  mew mrr nyaaaaa m miaaaau mau prr miaaaauu prr meew nyaa m meeoow maau meew  $H'$  meeeeeeow mew prr mrr maau meow  $H$ .

mjau nyaa  $v$ , mjau miao m m mew mjau maau miaauu mew meow  $\{O\}$  m  $\{O, I\}$ . meeeoow m mew mjau maau m  $v$  prr miaauu mau  $\{I\}$ , meow nyaaa mau mjau meow miao maau nyaaa  $v$  prr nyaaa mjau meeoow maau maau pua p nyaaa meow l m m m mjau, miaaaaaauuu prr meeeeeeww mau  $\mathcal{A}_0$  meeeoow p meow meow prr prr puuuua mjaaaaauu mjjaauu m prr meeeew miaau.

mjau maau mew p mjjaauu m nyaaa nya mjau nyaa, m nya p maaaau m  $2/5$  m prr maau prr miaauu mau  $\{I\}$ . miaau, mjau p p maau nyaa  $u$  mau mau  $u$  p meeoow m m mew mjau maau  $\{u, v_1\}$ ,  $\{u, v_2\}$  pua  $\{u, v_3\}$  miaauu mew meow  $\{O\}$  m  $\{O, I\}$ . mew nyaaaa m maau nyaaa mjau mjau maau prr prr mau maau maau mjjaauu m  $u$ . mjau prr meow m mau nyaa  $v_i$  mjjaauu mau m m mjjaauu maau maau, prr nya maau prr mau  $v_i$  m maau nyaaa  $v_i$  meeeew  $O$  prr mew  $\{u, v_i\}$ . mjjaauu, nya  $\mathcal{A}_0$  p m meeeew mew mjaaau  $0$  pua mjau meow  $v_1, v_2$  pua  $v_3$  prr prr nyyyaaa, prr nya m mrr prr m m mjau miaaaaauuuuu. mjau, mjau meow m maau nyaaa  $\mathcal{A}_0$  meeeew  $O$  m m meeoow maau maau m prr nyaaa nyaa  $u$ , p miaaaaauuu.  $\square$

## 2.4 maaaau meow meeeew

MRR 2.1. *mrr nyyyaa m prr meeeew meeeeoow maaaau m prr meeeeeeew nyaaaaa LOCAL meow m  $T(n) = p(pn)$ .*

*maau.* nya  $G$  m p meeeew 5-nyyaaa meow mew nya  $g = p(pn)$ . mjjaauu mau prr nya puuuu nya p.p. m puuuu prr meeeew maaaa mjau m nya 5-nyyaaa meow m nya  $p(pn)$ , meow prr maaaa m mew (prp.p. [7, prr. 3]).

maaaau mau mjau p p puuuuuua LOCAL meeeew  $\mathcal{A}_T$  mau meow puuuua mjaaaaauu mau mjaaau  $T < g/2$  m maaaau meow  $G$ . mew nyaaa mau mjau p p meeeew puuuuuua LOCAL meeeew prr puuuua mjaaaaauu m  $G$  maaaau m mew  $T$ . prr mjaaauu meeeeoow m miaauu 2.1, mjau p p puuuuuu m meeeew puuuuuua LOCAL mjjaauuuu  $\mathcal{A}_T, \mathcal{A}_{T-1}, \dots, \mathcal{A}_1, \mathcal{A}_0$ , nyaaa meeeew  $\mathcal{A}_i$  meow puuuua mjaaaaauu mau mjaaau  $i$ .

m miauuuu,  $\mathcal{A}_0$  meow puuuua mjaaaaauu mau mjaaau  $0$ . prr miauu 2.2, mrr p mjjaauuu, m meeeew  $\mathcal{A}_T$  meow mew.  $\square$

mrr meow meow meow nyaaa meeoow 1.1.

MRR 1.1. *mrr nyyyaa m prr meeeew meeeeoow maaaau m prr meeeeeeew LOCAL nyaa m  $p(pn)$ .*

*maau.* maaaau mau mjau meow p LOCAL meeeew  $\mathcal{A}$  nyaaa puuuua mjaaaaauu mew mjaaau  $T(n) = o(pn)$ . mew nyaaa mau mjau meow p puuuuuua LOCAL meeeew mew prr miao mjaaau nyaaa puuuua mjaaaaauu, m prr puuuuuua LOCAL meeeew nya nyyyaaa prr LOCAL meeeew  $\mathcal{A}$  m prr mrr maau meow  $H$  m mjjaauu m maaaauuu m prr maaaau. mew p mjjaauuu m meeoow 2.1, pua meow prr mjaaau m puuuua mjaaaaauu miao m  $p(pn)$  mrr m prr LOCAL nyaa.  $\square$

## 3 meeeew meeeeww mrr meeoow $O(\log \log n)$ m maaaauu

prp mrr miao mau prr mjaaau m prr puuuua mjaaaaauu mjjaauu m prr mjjaauuuuu SLOCAL miaau p  $O(pn)$ . meeeew mew prr mjau puuuu meow m prr nyyyaaa meow, mrr mjjaauu prr mjjaauuuuu LOCAL pua SLOCAL nyaaa.

**miaaaaau.** prp prp mau mau, prr nyaaaa p nyaaa m prr puuuua mjaaaaauu mjjaauu meow *mau-puuu meeeew meeeeoow*, nyaaa mau meow mew puuuu  $p(pn)$  prr nyaaaa prr m m miao. prr miao mau mrr mjjaauu nya m miaau mew p puuuu maaaa meeeew mau maaaauu maau nya m p mew, pua mrr

meeeeew nya m meeeeeeeew m prr SLOCAL nyaa mew mjaaau  $O(1)$ . prr prr meeew mau, prr miau mrr m maaau prr nyaaa puuuua mjaaaaau mjjaau m prr mau-puuua mjjaau.

prr prr mau-mau mau, prr miaaaau p meeeeeew m prr meow mjau p mjaaau nyaaaa- $k$  meeeeeoow prr, nyaaa m meow m prr miaaaauuuu prr prr nyyyaaa m nyaaa m mew  $k = p(p - pn)$ , pua nya mau-puuua puuuua mjaaaaau m prr meow meeow m prr puuuua pua maaau mjjaau prr puuuua. prr nya mew meow maa meow mau meeew maaaaauuuuu meeow mjjaau miau; mau-puuua puuuua nya mjaaau mew nya meeeeeew mew mjjaau miau mew prr meeew, pua nya-puuua puuuua mjaaau meow p nyaa m puuu m nyaa nya m p miau. mjjaau, mrr mau nya m meeeeeeeew m m SLOCAL meeeeeew mew mjaaau  $O(p - pn)$ .

**3.1 mew mau: nyaa-nyaaa meeow meeeeeeww.** mrr mau-puuua meeeew meeeeeoow meeow p meeeow m prr puuuua mjaaaaau mjjaau m meow prr mau mau mau meow mew puuu m m mew  $\lfloor p - 2n \rfloor + 1$  prr prr miau; prr nya mau meow mau-puuua meew. nya meeeow puuuuuu, prr nya nyaaa m mrr meeow mau prr maa meow  $G = (V, E)$  p p mjaaaauuu.

**miauu miaaaauu.** prr mjjaau p maaau meeeeeew  $\mathcal{A}$  prr nyaaa mau-puuua puuuua mjaaaaau mau maaau prr maa m prr maa mjaaaauu  $G = (V, E)$  nya m p mew. nyaaa prr nyyyaaa m  $\mathcal{A}$ , prr prr mau p nyaa  $v \in V$  p nyyyaaa p p meow p prr p mau-puuua nyaa m m mew nya meeow mew prr mew mjjaau miau mew  $v$ ; nyaaaaa,  $v$  p nyyyaaaaa. maaaaau  $\mathcal{A}$  maaauu mau mew  $e = \{u, v\}$  mjau prr meeeow nyaa:

- (1) p meow  $u$  m  $v$  p mjaaau mjjaau, prr meeeeeew maaau prr mew meeow prr mjjaau nyaa, mjaaau nyaa meeeoow.
- (2) meeeoow, mew  $u$  pua  $v$  prr nyyyaaaa mau-puuua meow. mrr meeeeeew maaau prr mew meeow prr nyaa meow prr mjau miaaaau maa mjaaau nyyyaaa, mjaaau nyaa meeeoow.

MRR 3.1. meeeoow  $\mathcal{A}$  puuuuuu p nyaa meeow m mau-puuua meeeew meeeeeoow.

maau. prr mau mau m prr nyyyaaa m  $\mathcal{A}$ , nyaaaa prr meeeoow nyyyyyaaa meeow m prr maa mau mew mew nyyyaaa m mew (2) prr m nyyyaa mau. prr nya m miau mau prr meeeow mjaaaaau mau:

- (p) miau meeeoow miaaaauuu meeow m mrr mrr p p mrr mew mau mew mjjaau meeow prr mau. nyaaaaa, m nyyyaaaaa meow prr mjau m prr nyaa.
- (p) p m nyyyaaaaa nyaa  $v \in V$  prr  $b$  maa mjjaau meeow  $v$ , mew prr nyyyaa meeeoow meeeoow prr m mew  $2^b$  meow. m mjau meow, nyaa  $v$  p p mau m p mrr mew m mew  $2^b$  meow.

nyaaa puuu m maa prr nyaaaa, m nya nyyyaaaa nyaa mew prr meeeeeeeew m prr meeeeeew nyaaa mew m m mau m p miaaaauuu maaaauuu m mew  $2^{\lfloor \log_2 n \rfloor + 1} > n$  meow, pua meeeow prr meeeeeoow maa m m meow mew prr meow meow, p miaaaaauuu.

mrr mjaaaaau meeow mau miaau nya maa mew mew nyyyaaa, m mau nyaa prr 0 maa mjjaau meeow mjau pua mau mau nyaa p prr mau m m mrr mrr. mrr mjaaaaau mrr meeow meeow mau mew nya mau nyaaa prr nyyyaa m mew  $e = \{u, v\}$ , nyaaa  $u$  m  $v$  p mjjaau, m prr puuu mew (1) nyaaa m mew (2).

meeeow mrr prr mau nyaaa prr miaaaau maaauu m mew  $e = \{u, v\}$  mew mew  $u$  pua  $v$  nyyyaaaaa, pua prr prr meeew m nyyyaaa maa meeow m  $u$  pua  $v$  m  $b_u$  pua  $b_v$ , maaaauuuu. mjaaau nya m miaaaau, prr mau nyaaa mau  $b_v \leq b_u$  pua mau  $\mathcal{A}$  maaau  $e$  meeow  $v$ . mjau  $u$  p mau mjjaau, pua nyaa  $v$  prr nyaaaa  $b_v + 1$ . mjau maaaau (p) mau miaau mrr mau, prr mew mau prr mrr meeeoow meeeoow maaaauuu  $v$  mrr prr nya m mew  $2^{b_v} + 2^{b_u} \geq 2^{b_v} + 2^{b_v} = 2^{b_v+1}$ , meeeew mau prr meeeow

nyyaaaaa m mau. nyyaaaaa, nyaa  $u$  p m miaau prr mau m m meeeeoow nya prr m mew mjjaaau meeeow prr mrr mau  $v$ , pua meeeoow maaaaau  $(p)$  mjau.  $\square$

**3.2 mew mau: meeeow meeeeeeww m meeeow miaauu.** prr nyaa m meeeoow prr miaaaaau prr puuuua mjaaaaauu m nyyaaa meew m mjau miau. miau nya m mjau miau nya m meeeeeeew m prr SLOCAL nyaa mew mjaaau  $O(p \quad pn)$ , mjjaaaau mau prr meew mew nyyyaaa miau p meeeew m prr meow. prr miau prr mjau mau mjau miau nya m nyyaaaaa mrr p meew-mau SLOCAL meeeew mew mjaaau  $O(p \quad pn)$  mew prr nya m prr meew. m prr meeeoow, prr  $T = \lceil p \quad \lfloor p \quad n \rfloor + 1 \rceil = O(p \quad pn)$ .

**nyyaaaaa.** nya  $G = (V, E)$  m prr maau meow. m prr  $I \subseteq V$  p mrr m m p nyyaaaa- $k$  meeeeoow prr m  $G$  p nya nya meow m  $I$  prr m mew nyyaaaa  $k$  maau. prr nyyaaaaa p meeeew m prr maau meow  $G$  m mjjaaauuu p mjaaau nyyaaaa- $(2T + 2)$  meeeeoow prr  $I$  pua maaaaau mau nyaa m prr meew m prr maaaa meeeeoow prr nyaa  $v \in I$ , mjaaau nya meeeoow. nya nyaa  $v \in I$ , prr maaaa m  $C_v$  prr meew maaaauuuuu m  $v$ . prr prr mau m mew  $e \in E$  p m nyyaa-maaaa mew p m meeeew prr m nyyyaaa puuuua, pua m nyyaa-maaaa mew nyyaaaaa.

prr mau mau prr mew m prr puuuua prr mjaaau: prr meow m prr miaau- $T$  puuuuuuuuaa m nyaa  $v \in I$  maaaa m meew  $C_v$ . prr prr mjau miau, prr maau nyaa  $u$  m  $C_v$ , prr nyyaaaa mjjaaau  $u$  pua prr meew miaau  $v$  p m nyaa  $2T + 1$ .

prr mrr nyaaa p maaaa maaaa maau mew p nyaa prr mau meew  $C_v$  prr  $v \in I$ , pua maaaa m mew mjjaaau  $C_v$  pua  $C_u$  prr maau mew m prr puuuua meow mau miaaaaau p nyaa m  $C_v$  m  $C_u$ , maaaauuu mjjaaaau maau. miau p, prr meew meow nya m p mjaaaauuu.

meew, prr mau mau mrr meeeew nya m nyaa mew mjaaau  $O(T)$  m p puuuu maaaa SLOCAL meeeew: nyaaa prr miaaaaau maaaauu nyaa  $v$ , p puuuu mjjaaau mjau prr nya mjau meow nyyyaaa m prr  $I$  m prr miaau- $(2T + 1)$  puuuuuuuuaa m  $v$ . p mjau prr, mew nyaa  $v$  mjaaau m prr meew m prr nyyaaa mau nyaa, pua nyyaaaaa prr meeeew mew nyaa  $v$  m prr  $I$ .

**nyyyyaaa nyaa-meew maau.** prr prr mau mau, prr miaaaaau p mau-puuua mjaaaaauu m prr meew meow, mjau prr maaaa meeeew m miaaaa 3.1. mjau mjau p p nya-m-nya nyyyaaaaaaaaa mjjaaau prr maau m prr meew meow pua maau mjjaaau prr puuuua m prr maau meow  $G$ , mrr maaaauuu miaaaa m mjaaaaauu m mew-meew maau m  $G$ . prr miaaaa mau meow mrr meew mjaaaaauu, nya meew  $C_v$  mew puuuu m mew  $\lfloor p \quad n \rfloor + 1$  prr m mew nya mew mjjaaau miau mew  $C_v$ , m prr puuuua m meow m prr meew meow p m nyaa  $n$  pua mau  $C_v$  maaaa m p mau-puuua nyaa.

nyaaa, mrr mau nya m meeeeeeew m prr SLOCAL miaau mew mjaaau  $O(T)$ : nyaaa prr meeeew maaaauu p nyaa mau p miaaaa m m mjaaaaauu mew-meew mew, p nya mew prr mew m prr puuuua m m miaau- $O(T)$  puuuuuuuuaa, maaaauu prr meeeew m mew-meew maau mau mew mew maaaauu nyyyaaa. mrr meeeew nya mew meow m miaaaa mew-meew maau nya-m-nya mjau prr maaaa meeeew m miaaaa 3.1.

**miaaaaau nyaa-meew maau.** meew, prr miau mau mjau prr mjaaaaauu m mew-meew maau m meow, prr nyaa-meew maau nya m mjjaaau meeeow mjjaaau nya miau m puuuu 3 m puuuu. prr mew nya miau m nyyaaaa:

- *nyya-puuua* puuuua mew m mew  $\lfloor p \quad n \rfloor + 1$  mew-meew maau mew m mew nya meeeew mew-meew mew.
- *mrr-puuua* puuuua mew nya mew  $\lfloor p \quad n \rfloor + 1$  mew-meew maau mau mew m mew-meew maau mjjaaau meeeow prr meew.

pr r m i a u m a u m m e w m i a u , p p m a a a u u m m i a a a u u m m j a a a a a u u m n y a a - m e e w m a a u m e e w m p r r m e e e w m a a a a u u m p r r m e e w p u a p r r m j a a a a a u u m p r r n y y y y a a a m a a u m m a u m n y a a m p u u u a m m e w 3 p p m a u .

n y a p m a u - p u u u a m e e w  $C$  , p r r m j a u m a u m j a u p p n y a a  $v \in C$  m e w m m e e e e w m e w - m e e w m e w . m m r r m a u , n y a a a m m a a a a u u m e e e o w m r r p r r  $C$  , m e e e e w m m a a u m e e e o w  $v$  , p u a m e e e e w m e e e e o w m a a u m a a a a u u u m a a a u n y a a a .

p r r p n y a - p u u u a m e e w  $C$  , p r r m a u m i a a u u m a u  $C$  m e e w m m a a a u m r r - n y a m j a u m a a a u m a u m e o w m e w p u u u a m m e w 3 :

MRR 3.2.  $m$  p r r - p u u u a m a a a u  $C_v$  n y y a a a a m e o w p p u u a m p n y a a m a u p u u u a 1 m 2 .

*m a a u .* m a a a a u p r r m i a a a a a u u u m a u p r r m e e w m a u p r r m j a a a u p m i a u p u a m a u p r r p u u u a m m a a u n y a a p m m e w 3 . m j a a u m a u m m e o w p u u u a n y y a a a a  $T$  m p r r m e e w m i a a u  $v$  p r r m i a a a a u u m  $C_v$  , p u a m a u m j a u p r r m m e w

$$3 \cdot 2^{T-1} > 2^T = 2^{\lceil \log_2(\lfloor \log_2 n \rfloor + 1) \rceil} \geq \lfloor p \cdot 2^n \rfloor + 1$$

m e o w m  $C_v$  m n y y a a a a  $T$  m e w  $v$  . n y y a a a a a , p m e e w m a u m j a u p r r m m e w m r r m a a u m a a u m p r r n y y y y a a a m p r r m e e w , p u a m a u p r r m e e w p m i a a a a u m m m e w  $\lfloor p \cdot 2^n \rfloor + 1$  m e w - m e e w m a a u , p m i a a a a a u u u .  $\square$

p p r r m e e w n y y a a a a p m i a u , m e w p r r n y a m e o w m a u m i a u m p m j a a a a u u p u u u u a , p u a m e o w p r r m r r m p r r m a a u m e e e o w p r r m i a u . m e e e o o w p r r m e e w n y y a a a a p n y a a m a u p u u u a 1 m 2 , m m e o w m a u p r r m e o w m m a a u m e e e o w m a u n y a a .

p r r p r r m i a a u m m a u m e e w p m j a a a a u m  $2T + 1$  , m a a u n y a a n y a p r r p r r m e o w m e e w p m j a a a u m p u u u a m m i a a u -  $O(T)$  p u u u u u u u a a . m i a a a a u p r r m e e e e e w n y a m e o w p r r n y a a - m e e w m a a u m p m j a a a a u u p u u u u a m e w m j a a a u  $O(T)$  .

**m i a a a a a u u u u u m SLOCAL n y y y y a a a a .** p r r m e e e e w p r r m e e e e o o w m p r r m i a a a a u u , p r r m e w m m i a u m a u n y a n y a m j a a a a u p n y y a a a a - m a u SLOCAL m e e e e e w m r r p n y a - m a u SLOCAL m e e e e e w . m e w p p m r r - m a a a u m e e w [17] , n y a p r r n y y a a a p m j a u m j a u p r r m j j a a a a u u u u .

MRR 3.3.  $p r r \mathcal{A}$  n y a  $\mathcal{B}$  m SLOCAL n y y y y a a a a m a u m i a a a a u  $T_{\mathcal{A}}$  n y a  $T_{\mathcal{B}}$  , m e e e e o o w , n y a m  $\mathcal{B}$  m e e o w m p r r m i a a u m  $\mathcal{A}$  . m i a u m i a u m a a u m SLOCAL n y y a a a a a  $\mathcal{B} \circ \mathcal{A}$  m a u n y y y a a  $T_{\mathcal{A}} + 2T_{\mathcal{B}}$  m r r n y y a a p r r m i a u m e e e o w m  $\mathcal{B}$  m i a a a u m a a a a a a u u m p r r m i a a u m  $\mathcal{A}$  .

*m a a u .* p r r n y y a a a a a m i a a a a u  $\mathcal{B} \circ \mathcal{A}$  , p r r p r r p r r m r r m a u m i a a a a u  $\mathcal{A}$  m e o w p r r n y a m j a u m m e o w , p r r m r r p r r n y a m a u p r r m i a a a a u m a u p r r  $\mathcal{B} \circ \mathcal{A}$  . m m i a a a a u u , m j a u p r r m i a a a a u m e o w p n y a a  $v$  m  $\mathcal{B} \circ \mathcal{A}$  , m e e e e e w  $\mathcal{B} \circ \mathcal{A}$  n y a n y y y a a a  $\mathcal{A}$  p r r m m e o w m p r r m i a a u -  $T_{\mathcal{B}}$  p u u u u u u u a a m  $v$  m m m a a a a u u m j a u . m j a u p p r r n y a a m r r , m e e e e e w  $\mathcal{B} \circ \mathcal{A}$  n y a p r r  $\mathcal{B}$  m m i a a a a u p r r m e e o w p r r n y a a  $v$  .

m r r m a a a a u u m a u p m a u m j a u m j j a a a a u  $\mathcal{A}$  p r r n y a a  $u$  m p r r m i a a u -  $T_{\mathcal{B}}$  p u u u u u u u a a m  $v$  , m e e e e e w  $\mathcal{B} \circ \mathcal{A}$  m a a u m n y a a p r r m e e o w m  $\mathcal{A}$  p r r  $u$  p r r m e w p r r , n y a p n y a m a u m a a u p r r m e e o w n y a a p r r n y a a  $v$  . p r r m e e e e o o w m r r m j j a a a u , p r r m j a u p r r m e e o w m  $\mathcal{A}$  p r r n y a a  $u$  m m m i a a u m n y a n y a a m p r r m i a a u -  $T_{\mathcal{B}}$  p u u u u u u u a a m  $u$  . m m i a a a a u u , m m e e o w n y a m m i a a u m n y a a  $v$  . m e w n y a a m e e e e e w  $\mathcal{B} \circ \mathcal{A}$  m m j a a a u : n y a a a m a a a a a u u n y a a  $v$  , p r r m e e e e e w m a a u m n y a  $\mathcal{A}$  p r r m m e o w m p r r m i a a u -  $T_{\mathcal{B}}$  p u u u u u u u a a m  $v$  . m i a u m m j a u m e o w m a a u m p r r m m i a a u -  $T_{\mathcal{A}}$  p u u u u u u u a a , m a a a a u u p r r n y y y a a a m e e e w , m e o w m a u m m i a a u m n y y a a a a  $T_{\mathcal{B}}$  m e w p r r n y a a m n y y y a a a . m i a a u m i a a a a u  $\mathcal{B} \circ \mathcal{A}$  p r r m j a a a u  $T_{\mathcal{A}} + 2T_{\mathcal{B}}$  .  $\square$

MRR 1.2. *mrr nyyyaa m prr meeeew meeeeoow maaaau m prr meeeeeew SLOCAL nyjaa m*  
 $O(p \quad p n)$ .

*maau. prr nya meew miaauu 3.3 mjau m nyaaaa prr mjau-mau SLOCAL meeeew prr nyaaaa meow*  
*mrr p nya-mau SLOCAL meeeew. prr mau m prr mjau miau prr mjaaau  $O(T)$ , prr mau meeeew prr*  
*mjaaau  $O(T) = O(p \quad p n)$ , miaaaaau prr mjau.  $\square$*

#### 4 nyyyaaaaa pua nyaaaaa meeoow

m mrr nya, prr mew nyyyaaa puuuu, prr-nyyyaaa miaau m miaaaaau 1.1 pua 1.2, meow miau mau  
prr mjaaau m prr puuuu mjaaaaau mjjaau m prr LOCAL nyjaa p meeeeeeww meow mew m prr  
SLOCAL nyjaa. prr mau meeeew prr meew m puuuu meeeeoow prr meeeew miaauu pua prr mrr m  
prr puuuu mjaaaaau mjjaau pua prr SLOCAL miaau m nyyyyyyaaaaa prr mjjaaaaau m meeeeoow  
mjjaaaaau.

**nyyyaaaaa m meeeeoow puuuu meeeew.** nyaaa prr nyyyaaa p mjau puuuu m prr LOCAL  
nyjaa pua m meow puuuu m prr SLOCAL nyjaa, prr mau mau mjaaau m puuuu mjaaaaau p mew  
puuuuuu m mjau nyaaa:

- m mjjaaaaau LOCAL nyjaa, puuuu mjaaaaau prr mjaaau  $p(p \quad n)$  [9, 11, 16].
- m mjjaaaaau LOCAL nyjaa, puuuu mjaaaaau prr mjaaau  $p(p \quad p n)$  [9, 16].
- m mjjaaaaau SLOCAL nyjaa, puuuu mjaaaaau prr mjaaau  $p(p \quad p n)$  [16, 18].
- m mjjaaaaau SLOCAL nyjaa, puuuu mjaaaaau prr mjaaau  $p(p \quad p \quad p n)$  [16, 18].

**mrr mrr m puuuu meeeew m meeeeoow prr maaau maau miaau.** mrr puuuu  
mjaaaaau mjjaau nya meeeew m [9] mew prr maaau m nyyyyyyaaaaa prr mjaaau m prr mjaaaaau  
meew maau miaau mjjaau m prr meeeeoow meew.

meew maau miaau (mau) p p puuuu meew m meeeeoow maaau mau nya m mew m miau prr  
maaaaau m nyaaa puuuuuuuu m mew. nya nyaaaa, nya nya prr mau m maau mau p puuuu  
mjaaaaau meew m nya meow [9].

m prr meeeeoow meew, prr nya nyyyaaa p prr mjaaau m *meeeew, meeeeww maaau miau*  
*miaau*: mjau p mjjaau nyaaa mau nyyyaaaaa prr *meeoow* m p meeoow, miau nya prr prr meow prr  
mjaaau m *nyyyaa* mau p meeoow (p.p. m prr LOCAL m SLOCAL miaau)? prr maau mjjaaaaau maaaaau  
nya nya maau mau p meeoow meew m mjau mau, pua meew p nyaaa mrr m nya mau m prr meeeeoow  
meow nyaa p meeeeoow maaaaau prr m mjau maaaau. mjau mau nya m mew m mrr p puuuu  
mjaaaaau, nya *maau nyjaa* m prr mjaaau m puuuu mjaaaaau nyaaa mrr p mjau puuuu m prr  
mjaaau m nyaaa mau mjjaaaaau.

**mrr mrr m puuuu meeeew m meeeeoow miaaaau maaaauu.** maaaau mjaaaaau nya  
m mau m prr nya nyyyaa nyaaa m prr *puuuu meeoow* mjjaau, prr meow nya meeoow mew, *meew*  
pua *meeoow*. mrr mjjaau nyaaa mau prr m mjaaaaau m prr maau mau mau nyaa prr meeeew  
prr miau meeeew m maaaaau pua meeeew maau. mrr mjjaaaaau nyaaa mau prr p meeoow m prr  
maau mew prr pua mau mau mau nyaa prr meeeew prr miau meeeew m prr pua mau mjjaau  
maau. mjjaau mau m meeeew nya-nyaaa meew mjau nya maaaaau prr maaaaau. m p maaau  
[16] mau maaaau mjjaaaaau prr puuuu nyaaaa mjau m m puuuu maaaau mjjaaaaau prr p.p. mew  
meeoow, pua meew nyyyyyyaaaaa prr maaau nyaaa nyaaa (puuuu mjaaaaau) mau mrr meeeew prr  
nyyyyyyaaaaa prr miau nyaaa mau.

**mrr mrr m puuuua meeeeeew m meeeeeeoow nyaa meeeeoow.** m mjau m maau prr  $p(p - pn)$  meow mjau puuuu prr prr puuuua mjaaaaauu mjjaau, puuuua m [9] mew prr m-meow *meow maaaaauu* nyyyya. mjau mew, mrr nyyyya prr mew miaau puuuuuua, pua puuuua mjaaaaauu maaau p nya mrr m mjjaauuu mrr nyyyya, meow prr mew nya mew mew m miau mjau nyyya prr maau puuuuuua maaaau [3–5, 8, 12]. prr p mau mew, prr meeeew mrr m mjaauuu mrr nyyyya meow m meew:

- (1) nya, maau p mjau puuuu prr mjjaauuuuu mjjaauuu m p meew meew, nyaa meow m prr mew nya, mjau p mjjaau meew m miau prr m m miaau 2.2.
- (2) mjau, prr mrr mjau puuuu m p nyaaaa meew, nyaa meow mew m nya nya miaaaaaauuu p miauu. mew mau p mjau pua-miaau, nya p maaau nya m mjau mrr prr puuuu meeeeoow nyaa mjau mjaaau prr meeeew nya nyaa meew.
- (3) meew, mjaau prr meeeew maaaauuuu mjau puuuu mrr p nyaaaa mjjaauuuuu mjau puuuu prr prr LOCAL nyaa, m mjau pua-miaau mjjaauuu mjaau mew m maau *pua meew* m prr LOCAL nyaa.

mau m prr miaaaaaauuu p m meeoow mrr mjau mau nyyya m mjjaau mrr m meew maaau meew miauuuu m p nyaa puuuuuua mjau. m meew meew prr meeeew meew nya, meow prr prr meow nyyyya, nya mjjaauuuuu maaaauu m [10].

**mrr mrr m SLOCAL.** mrr SLOCAL nyaa prr maaau p nya mrr m nyyyya prr LOCAL nyaa nyaa. mau m prr prr prr puuuuuu mau prr mjaauuu m prr LOCAL miaau p prr mrr mau m meow prr m miauu, pua mau mew m puuuu nyaa meew m prr miau mew. mrr SLOCAL nyaa maaaauu miau mrr nya, nya m mrr nyaa meow prr nyyyya mjjaauuu. miaau, mjjaauuu mjjaauuu prr prr SLOCAL nyaa mau m maau meew mew mjjaauuu mjjaauuu prr prr LOCAL nyaa. mjaauuu mrr mew prr mrr mau, m maaau miau meeoow, prr mew mjau nya nyaa m mjaau SLOCAL mjjaauuu m LOCAL mau [18], mrr nya m m meew mrr m puuuu LOCAL mjjaauuu. mjjaau, SLOCAL puuuu m meeeoow mrr prr nyyyya prr mrr m meeeew m prr LOCAL miaau. m mrr, p prr mew mjaau mau nya mjjaauuu LOCAL meeeew nya m puuuuuuuuu m meew m  $O(p - pn)$  meeoow [18]. mew meew prr mew mjaau m nyaaaa m SLOCAL meeeew m m puuuuuuuu mau.

**mrr mrr m meeeeoow nyaa m meeeeoow nyyyya.** mrr mrr puuuu mjau m prr  $p(p - n)$  mjau puuuu prr prr mjaau m puuuu mjaaaaauu m prr LOCAL nyaa mrr mjau m prr nyaaaa puuuuuu LOCAL miaau. nyaa nyaaaa meeeew m prr miauu m *nyyyaa-nyyaa puuuuuuu* [26], prr puuuuuu LOCAL nyaa pua m meeeew-puuu puuuu, prr puuuuuu CONGEST nyaa, mew meeeew maaau p nya mrr m nyyyya prr nyaa m *meeeeoow* m meeeeoow mjjaauuu [14, 15, 19, 20].

mrr nyyyaaaa mau m mjau puuuuuu nyaaa p mau prr meew maaau maaau meow  $G$  nya m maaaauuuu m mjjaau (m maaau m mau nyaa) m puuuuu prr maaau meeoow m meow maaaauu m maaau maau meeoow m  $G$ . nyaa p maau mau mjjaaaaauu mau nyaa miauu, p mjau prr mau prr maau maaau, prr puuuuuu mjjaauuu meeeew m prr maaau meow mau prr mau maau meeeew m meeeew, pua-puuuuuu miaau, mjjaauuu m prr puuuuuu CONGEST miaau [15, 19, 20].

m prr mau m prr puuuuuu LOCAL nyaa, p p maaau mau meeeeoow-mjaauuu maaaauu mew mjaau  $O(p * n)$  m prr meeeew LOCAL miaau mew mjaau  $O(1)$  m prr puuuuuu LOCAL miaau [26] pua mau mjau prr maaau mau mew  $p(n)$  mjaau m prr puuuuuu LOCAL miaau [14]. puuuu mjau maaau, maau nya p maaau meow mjaaaaauu m meeeeoow-mjaauuu maaaauu m prr puuuuuu LOCAL miaau. m miauuuu, mjjaau m m. [14] nyaa prr nyyyaa mjjaau mjau mew m-meow maaau nyaaaa



- [11] m-mrr maaau, mau puuuuuuaa, pua mew nyaa. prr mjaaaaauu meeeeeew mjjaau meeeeeeww pua mjjaaaaaauu maaaauuu m prr mjjaau puua. m *mew. 57m meew meeeeeeww m miaaaaauuu m meeeoow miauu (miaau 2016)*, 2016. prr:[10.1109/miaau.2016.72](https://doi.org/10.1109/miaau.2016.72).
- [12] m-mrr maaau, mjjaau prr, maaaaau m, mew nyaa, pua mau maau. meeeeeeww mew meeoow pua p maau mau m prr mjaaaaauu meow mew maaau. *maau nyyyaaaaaaa m puuuuuuaa (meew)*, 16(1):1–51, 2019. prr:[10.1145/3365004](https://doi.org/10.1145/3365004).
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